

Fundamental Uncertainty in Model Predictions: Analysis of Modern Macroeconomic Models from the Perspective of Friedman's Instrumentalism

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Abstract: In 1966, M. Friedman published an essay on the methodology of positive economics, in which he emphasized the role of predictions as a decisive criterion for accepting macroeconomic models. This paper analyzes to what extent modern macroeconomics in 2023 is guided by these ideas. The first part of this paper deals with the role of predictions in macroeconomic models and shows that a large part of the models currently used do not contain any predictions at all. The next part of the paper explains the problem of Lucas' critique and the resulting complications in making predictions. The following section shows how modern dynamic stochastic general equilibrium (DSGE) models are attempting to deal with Lucas' criticism. The last part of the text analyzes further fundamental complications in making macroeconomic predictions, such as the issue of free human decision-making, the butterfly effect and the normative form of the questions being studied. The paper shows that a portion of modern macroeconomic models that do not contain predictions would undoubtedly fail Friedman's test, and models created for the purpose of prediction still have not completely dealt with serious methodological complications. Due to the nature of the subject matter, macroeconomic models will likely never be able to provide completely reliable predictions.

Keywords: Methodology, Granger causality, Lucas critique, Forecasting, Butterfly effect.

1 Introduction

In 1966, M. Friedman published his essay on the methodology of positive economics, in which he emphasized the crucial role of predictions in accepting macroeconomic models. According to Friedman, the model's ability to predict the phenomena it is intended to predict is the decisive factor in evaluating it, and the only test of the hypothesis's validity is comparing its predictions with experience. In his text, Friedman thus downplays the importance of the realism of assumptions, and the only decisive factor in accepting assumptions is the model's good predictive ability as such. Friedman thus joins the so-called instrumentalists who take the view that theory is primarily an instrument for predicting observable reality in science². This text will examine to what extent modern macroeconomics in 2023 follows this Friedman's ideas.

The first part of this paper will deal with the role of prediction in macroeconomic modeling. The next part will compare macroeconomics and natural sciences. Since one of the differences will be the instability of the parameters used, the next part will focus on Lucas's critique and its subsequent resolution using dynamic stochastic general equilibrium models. The final part of

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² Friedman (1966), summary and analysis of his ideas in Wong (1973).

the paper will address other fundamental factors that explain phenomena complicating predictions, and which likely even exclude reliable forecasts. The conclusion then summarizes the findings presented.

2 The Role of Forecasting in Macroeconomic Modeling

The very concept of forecasting in macroeconomic models is not straightforward, and when specified more precisely, we encounter a number of complications. This chapter will show that there are generally two different types of forecasting in macroeconomic models, and that a significant portion of macroeconomic models does not contain any forecasting at all.

The first type of prediction is the definition of a lead-lag relationship, where based on information at time t , we are able to predict the development at time $t+1$ ³. Coincidentally, two years before M. Friedman's aforementioned essay (1966), a paper describing the still key methodology for capturing the predictiveness of relationships was published, which is the concept of Granger causality (in macroeconomics it began to be widely used in the 1980s⁴).

Granger causality analyzes the predictive relationship between variables X and Y , where variable X at time t influences variable Y at time $t+1$ ⁵. To precisely understand the method of examining this relationship, let us define the information set I at time t as variable I_t , which contains all available information, and let $I_{-X,t}$ denote the same information set without variable X . Granger causality can then be expressed as follows

$$\mathbb{P}[Y_{t+1} \in A \mid I_t] \neq \mathbb{P}[Y_{t+1} \in A \mid I_{-X,t}] \quad (1)$$

The left-hand side of equation 1 shows the probability with which we are able to determine the value of Y at time $t+1$ based on the information set I_t . The right-hand side shows this probability with the information set I_{-X} , which does not contain information about variable X . If the mentioned inequality holds, then we are able to determine the value of Y_{t+1} with a higher probability when we have knowledge of past values of variable X , and thus variable X enters into Y causally. The validity of the mentioned inequality can be determined with a specific p-value, confirming or refuting Granger causality at the probability level that we have determined.

The second type of forecasting is a relationship that takes the form of a standard linear regression without the presence of historical variables⁶. This involves forecasting the current state based on known current variables. A typical use case is nowcasting, where subsequently published variables (typically GDP or inflation) are forecast based on currently available data (electricity consumption, traffic, etc.)⁷. Let Y be a variable whose current value can be explained by the explanatory variable X , based on the relationship

$$Y_t = a + \beta X_t + u_t \quad (2)$$

³ We will continue to use the standard time operator t for variables, where t represents the current period, $t-1$ represents one period ago, $t+1$ represents one period ahead and so on.

⁴ Maziarz (2015).

⁵ Granger (1969).

⁶ We will address why we in this case are talking about forecasting rather than just estimation later in this section.

⁷ As an example, let us mention the Rushin model constructed by the Czech National Bank (Adam et al. (2021)), which uses, among others, values of electricity consumption, internet searches and the amount of toll paid on Czech highways to estimate the current GDP.

The relationship can be further extended to include more explanatory variables. Let Y be the dependent variable and $X1$ and $X2$ be the explanatory variables. The linear regression then takes the form

$$Y_t = a + \beta_1 X1_t + \beta_2 X2_t + u_t \quad (3)$$

The explained current value of the variable Y is in this case a function of the current values of the explanatory variables $X1$ and $X2$. In other words, based on the current values of the explanatory variables, we predict the current value of the explained variable. The coefficient of determination provides information about how much of the variation in Y is explained by the values of $X1$ and $X2$. This coefficient, on a scale from 0 to 1, indicates the percentage of Y 's movement that can be explained by the chosen explanatory variables. Let's introduce the variable \hat{Y}_t , which represents the estimate of the variable Y purely based on the variables $X1$ and $X2$, and further introduce the variable \bar{Y} as the mean value of the variable Y .⁸ The coefficient of determination can then be expressed by the equation

$$R^2 = \frac{\sum_{t=1}^T (\hat{Y}_t - \bar{Y})^2}{\sum_{t=1}^T (Y_t - \bar{Y})^2} \quad R^2 \in (0,1) \quad (4)$$

If linear regression is successful in predicting using equation 2 or 3, it means that we understand the system (we know the variables that can explain the behavior of variable Y , up to the level of the coefficient of determination⁹).

In the case of nowcasting, it is acceptable to acknowledge the fact that it is a forecast, because we are predicting later officially published true values. However, a large part of the currently used macroeconomic models do not forecast at all, because the explained variable is not subsequently specified. Based on current variables, other current variables are estimated, which due to their abstract nature are not confirmed in the future.

As an example, consider the widely used equilibrium exchange rate model BEER. In this model, the current equilibrium exchange rate is a function of the current value of GDP (gdp_t), net foreign assets (nfa_t) and terms of trade (tot_t)¹⁰, and can be expressed by the equation

$$rer_t^{BEER} = \gamma + \alpha_1 gdp_t + \alpha_2 nfa_t + \alpha_3 tot_t \quad (5)$$

Based on current values of the explanatory variables, we estimate the current value of an abstract variable that is not later confirmed. If we were to conclude that this model cannot be used within the framework of Friedman's methodology due to the absence of forecasting, we would lose a model that currently provides good results in this area¹¹.

⁸ For a higher number of explanatory variables, an adjusted coefficient of determination is used, which takes into account the penalty for each additional explanatory variable.

⁹ The proper construction of linear regression is more complex and, in addition to stationarity, also requires analysis of autocorrelation, multicollinearity and other factors.

¹⁰ Zorzi et al. (2020).

¹¹ The BEER model is one of the most commonly used models in both the private and public sectors, along with purchasing power parity models and Natrex. For more information, see Zorzi et al. (2020) and Mandel & Tomšík (2009).

Regarding the incorrectness of rejecting these models in macroeconomic modeling, let us present the following thought experiment. If, in the case of the BEER model, we wanted to obtain the first type of prediction based on leading relationships, then instead of current variables gdp_t , nfa_t and tot_t , we would use their historical development. By using their historical variables instead of current ones, the model would demonstrably have lower predictive power, and in order to adhere to the described Friedman's methodology, we would knowingly use a worse model¹².

3 Formalization of Lead-lag Forecasting

To provide a more precise description of forecasting in macroeconomic models, let us state that the second type of forecasting described above is only possible when the linear regression (equations 2 and 3) contains variables only at time t . If historical variables are included in the linear regression, it becomes a leading relationship captured by Granger causality, and thus it becomes the first type of forecasting.

To capture the relationship using historical variables, vector autoregression models can be used, in which the development of variable x depends on its own historical development and the variable y . This relationship goes back to a history of length P , whose optimal value is determined using information criteria.

For simplicity, let us consider a system of length $P=1$. Let us define the coefficient $a_{1,1}$ for the influence of the past value of x on the current value of x , and $a_{1,2}$ for the influence of the past value of y on the current value of x . Similarly, let us define $a_{2,1}$ for the influence of the past value of x on y , and $a_{2,2}$ for the influence of the past value of y on the current value of y . Then the dynamic system can be expressed using the following matrix notation

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix} \quad (6)$$

From equation 6, it is clear that in the case of significant parameters a , Granger causality is present in the system. For a general length P , we again obtain a linear regression, but now with lagged variables, in the form of

$$x_t = c_1 + \sum_{i=1}^P \alpha_i x_{t-i} + \sum_{j=1}^P \beta_j y_{t-j} + u_t \quad (7)$$

The fact that equation 7 only contains lead relationships allows the system to be described by Granger causality, which takes into account the influence of all historical variables y up to length P on the current value of x . The absence of Granger causality (in our case, the null hypothesis) can be confirmed after estimating the linear regression in a situation where all β coefficients are equal to zero (see equation 8), which means that historical values of y do not enter into the current value of x in any way.

$$H_0: \beta_1 = \beta_2 = \dots = \beta_p = 0 \quad (8)$$

¹² However, the question remains of how to confirm the quality of the estimate, as the accuracy of the estimated variable cannot be verified and is dependent on the fundamental construction of the model.

Based on this description, it is clear that Granger causality only indicates the presence of a predictive relationship between variables. Granger causality does not necessarily imply causality in the traditional sense. It is possible that variables x and y causally respond to an unobserved variable z , with y only responding more quickly. Even when detecting Granger causality between variables, it is therefore necessary to further fundamentally analyze the variables involved¹³.

In addition to the two types of prediction mentioned, there are other concepts that lie at their border. A significant concept in the field of applied macroeconomics and physics is the so-called Kalman filter. This method takes into account the present data, which it compares with historical data. If we consider the estimate of the current position of a rocket, we have currently transmitted data available. Since these data exhibit noise, the Kalman filter also takes into account their previous development and estimates the subsequent motion based on natural laws from historical data. By combining the estimate based on historical data and information from current data containing noise, we obtain a more accurate prediction of the rocket's position than when using only one method. The Kalman filter thus combines both the first and second types of predictions¹⁴.

In his essay, M. Friedman undoubtedly had the first type of forecasting in mind (as also indicated by the structure of his work, such as Friedman & Schwarz (1963), in which he deals with lead-lag relationships). As the chapter has shown, models that do not predict still exist in modern macroeconomics. Moreover, current macroeconomic research utilizes both types of forecasting, where economic phenomena are predicted based on historical variables' influence on current values and the interrelationship among current values.

4 Predictability and Natural Science

Friedman's call for positive economics as a method of studying social phenomena using the methods of natural sciences was not new in the 1960s. This concept can be traced back to the founders of modern macroeconomics such as J. M. Keynes or P. Samuelson¹⁵. The success of the natural science methodology was undeniable in the 1960s, and efforts to build on the successes of natural science methodology in other areas of research were therefore a logical outcome.

¹³ To illustrate, let's use an example of a topic analyzed by M. Friedman, who studied the relationship between the quantity of money and inflation. Friedman & Schwarz (1963) pointed out a leading causal relationship, where the creation of money causally leads to inflation (the so-called Friedman monetarism). If we decide to test this hypothesis using Granger causality, its confirmation does not mean the confirmation of a truly causal relationship. If we take into account some modern theories of endogenous money, then inflation and changes in the quantity of money are a function of economic growth. If the economy grows, new money is created endogenously, and at the same time, increasing aggregate demand leads to inflation (the so-called NK IS curve). In such a situation, we can assume that money is created endogenously faster than inflation reacts. Thus, the data will show Granger causality between the quantity of money and inflation, but under the validity of the mentioned theories of endogenous money, both variables just react at different speeds to strong economic growth.

¹⁴ The mathematical construction of the Kalman filter itself is quite complex and its detailed description can be found, for example, in Sargent & Stachurski (2022), while its use in macroeconomic forecasting can be found in Pasricha (2006).

¹⁵ Samuelson (1964). Although P. Samuelson wanted to construct macroeconomics as a positive science, he had strong objections to several points in M. Friedman's essay. More about these objections and his arguments can be found in Wong (1973).

In natural sciences, the ability to make predictions is a necessary condition for accepting a theory. Let's use two examples to explain this fact. The first example is the general theory of relativity. A. Einstein established several specific predictions that would confirm his theory. These included the bending of light during a solar eclipse caused by its gravitational field, which bends the direction of photons when traveling past distant objects. It also included the precession of the perihelion of the planet Neptune due to the specific effects of the theory, or the gravitational waves emitted when very massive objects are accelerated¹⁶. These and several other predictions specific to the general theory of relativity were confirmed by subsequent experiments, and the theory thus became generally accepted¹⁷. As for the realism of the assumptions, the general theory of relativity does not explain phenomena, but only describes the curvature of space and its impact on time and the motion of objects. Acceptance and construction of this theory are thus in complete accordance with Friedman's ideas.

When it comes to the role of assumptions in process of hypothesis acceptance in natural sciences, let's illustrate this procedure on the so-called theory of superstrings. The theory is based on solidly established assumptions, but it has not yet been generally accepted. The theory makes its own specific predictions, which have not yet been confirmed¹⁸. Since the ability of the theory to confirm its specific predictions has not yet been confirmed, the hypothesis has not been accepted, despite its solid theoretical foundation. These selected examples show that natural sciences proceed according to the key described by M. Friedman, and a theory is accepted only when its ability to predict specific future events for the theory is demonstrated¹⁹.

M. Friedman states that accepting models based on predictions is complicated in natural sciences due to external factors, and he illustrates this fact with a model describing free fall, in which the distance traveled by an object over time can be described by the following equation

$$s = 1/2gt^2 \quad (9)$$

where s represents distance in feet, t represents time in seconds and g represents a unchanging physical constant. If we confirm this relationship with predictions during the free fall of a heavy ball, we will undoubtedly accept the hypothesis from equation 9. However, if we perform the same experiment with a light feather, the model will not provide the correct prediction due to external influences, such as air resistance and so on. Similar analogies can be found in macroeconomic models, which do not provide correct predictions despite their correctness, precisely due to interconnected external factors. The same argument is sometimes heard in meteorology (which we will address in chapter 9), where, despite knowledge of fundamental laws, correct predictions are generally impossible.

The essential thing about the described physical models is that their equations contain stable parameters. The mentioned theory of general relativity works with the gravitational constant, which is predefined and unchangeable for our universe. Similarly, for our universe, the equations involve constant and unchanging speed of light, Planck's constant or our physical constant g

¹⁶ There are significantly more specific evidence of the theory of general relativity, for a detailed list refer to Ferreira (2014).

¹⁷ Weinstein (2015).

¹⁸ This includes, among others, supersymmetric particles or the observation of extra dimensions in accelerators. For more information, see Greene (1999).

¹⁹ The description of the methodology of physics and natural sciences is, of course, only indicative and simplified to highlight the ideas presented in this paper. There are long-standing disputes among physicists about methodology, and more on this issue can be found in Radder (2009).

from equation 9. This fact is crucial for applying these methods to macroeconomic modeling. As the next chapter will show, macroeconomics cannot rely on such stable parameters, which significantly complicates the construction of its models.

5 Lucas Critique

The fact that macroeconomic models cannot rely on stable parameters leads to problems especially in forecasting. If the parameters used in forecasting are not stable, their value in the forecasted period may no longer hold.

To illustrate this problem, let's use the popular Okun's Law from the 1960s. It states that the difference between GDP and its potential is a function of the distance between unemployment and its natural rate, and this relationship can be expressed as follows

$$Y^* - Y_t = c(u_t - u^n) + e_t \quad (10)$$

Where Y^* represents potential output, variable Y_t represents output at time t , variable u_t represents unemployment at time t and u^n represents natural rate of unemployment²⁰. The problem with modeling arises not only in a single case, namely in linear regression with knowledge of variables Y^* , Y_t , u_t and u^n . In such a case, the coefficient c can be determined, and the result can be summarized by stating that at time t , unemployment entered the deviation from potential output with a size of c , and the size of variable e_t is the detected influence of other factors.

The problem with the second type of prediction arises when we know the variables Y^* , u_t and u^n , and we do not know the variable Y_t . In such cases, we must assume a stable coefficient c implicitly. The same problem also arises in the first type of prediction, where we again have to rely implicitly on a stable coefficient c . If its value changes, even the first type of prediction does not provide accurate results.

This fact and its disregard led to the so-called Lucas critique. Lucas (1976) argues that when predicting based on macroeconomic models, parameters whose stability is not fundamentally ensured cannot be used²¹. This concept was widely accepted and led to the development of a new concept of macroeconomic models, nowadays known as DSGE²². The development of these models was motivated by the pressure to address the Lucas critique and to improve the models' predictive ability, with the aim of approaching the methodology of natural sciences. There was also demand from the private and public sectors for high-quality and reliable forecasts.

²⁰ For the sake of simplicity, let us consider variables Y^* and u^n to be constant.

²¹ Although it might seem from the economic literature that it was a groundbreaking insight at the time (year 1976), in reality, this fact was evident, and economists were aware of it in previous periods. As an example, we can mention the addition to the Phillips curve by M. Friedman, who identified the unstable parameter that shows the strength of the relationship between inflation and unemployment as the reason for its incorrect predictions (Friedman (1968)). He solved this issue by replacing the original static expectations $E_t(\pi_{t+1}) = \pi_t$ with adaptive expectations, which can be expressed as $E_t(\pi_{t+1}) = a\pi_t + (1-a)\pi_{t-1}$, where π represents inflation and E_t is the expectation operator at time t . Although this step improved the fit of the Phillips curve to empirical data, the problem of Lucas' criticism in the model was not resolved and remained in the mentioned parameter a . However, the discussion on the stability of parameters dates back much earlier, and these problems were already addressed for example by Frisch (1938).

²² The abbreviation DSGE stands for Dynamic Stochastic General Equilibrium Models.

6 Macroeconomic Modeling After Lucas's Critique

As a response to the Lucas critique, a new concept of DSGE models was developed, initially called the RBC model²³. Lucas's critique demanded that new models not rely on unstable macroeconomic coefficients and naive backward-looking formation of expectations²⁴. The concept of RBC models addresses this requirement by constructing models based on microeconomic foundations and analyzing macroeconomic developments through the decisions of individual agents.

Unstable macroeconomic coefficients are replaced by microeconomic coefficients describing the psychological characteristics of people, which can be identified through microeconomic analysis. At the same time, rational expectations are built into the models, where economic agents actively predict the future based on the information available to them at a given time.

To illustrate these ideas, let us describe the basic structure of this model. Individual agents decide on the amount of consumption (C_t) and the amount of worked hours N_t , while deciding between work and leisure time ($1=L_t+N_t$, where the variable L_t represents the leisure time that brings utility to the agent). The current utility function $U(C_t, L_t)$ can be written as follows

$$U(C_t, L_t) = \frac{[(C_t^\mu L_t^{1-\mu})^{1-\sigma}] - 1}{1 - \sigma} \quad (11)$$

Where the variable μ represents the amount of labor supplied based on the decisions of the agents, and the variable σ represents the microeconomic coefficient for relative risk aversion. The total utility that agents try to maximize is composed of the expected discounted sum of current utilities, and can be described by the equation

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \quad (12)$$

Where the operator E_0 represents expectations at the initial time 0 based on the information set available at that time, and the variable β indicates the discount rate. The variable β is endogenized in the form $\beta=1/(1+\rho)$, where ρ represents the subjective discount rate.

We currently have the coefficients σ and ρ (while the coefficient μ in the model is fully endogenized). Although these coefficients do not yet have the form of stable parameters, they differ significantly from the coefficient c in equation 10. These are coefficients that result from microeconomic decision-making by individual agents, using forward-looking rational expectations. The size of these coefficients can be determined based on applied research in human psychology or microeconomic analysis²⁵. The depth of their endogenization determines to what extent the models will be immune to Lucas's critique.

²³ The abbreviation RBC stands for Real Business Cycle.

²⁴ Lucas (1976). As Lucas critique, we will consider all unstable parameters that complicate the functioning of the model. To explain the precise meaning of the original concept in Goutsmedt et al. (2016).

²⁵ The coefficient σ has been further studied in Rotschedl (2015). The stability and value of the ρ based on psychological characteristics are examined in Praag & Booij (2003). Endogenization of this parameter is discussed in Schumacher (2006).

The next equation in the model is the capital accumulation equation, which states that the capital stock in period $t+1$ (K_{t+1}) is equal to the capital stock in period t (K_t) decreased by depreciation (δ) and increased by investment (I_t) made in period t . This relationship is captured by the following equation

$$K_{t+1} = (1 - \delta)K_t + I_t \quad (13)$$

Parameter δ appearing in equation 13 represents an empirically determinable variable describing the rate of capital depreciation between periods. In addition to direct methods, such as analysis of company statements, more sophisticated modeling approaches can be used, such as the perpetual inventory method²⁶. Therefore, this parameter is not completely immune to Lucas's critique, because the stability of its value must be implicitly assumed for the future.

The producing sector in the model consists of firms that maximize their profit π , whose size is the result of the following equation

$$\pi_t = Y_t - w_t N_t - r_t K_t \quad (14)$$

where variable Y_t represents production, w_t represents the wage rate and r_t represents the interest rate, which shows the cost of capital. The wage rate equals the marginal product of labor and the interest rate equals the marginal product of capital. Both of these variables are therefore fully endogenized within the model.

The final key building block is the production function, which captures the way in which firms produce output Y_t based on inputs of capital K_t and labor N_t , and can be expressed by an equation

$$Y_t = A_t (K_t)^\alpha (X_t N_t)^{1-\alpha} \quad (15)$$

The influence of capital on the size of output is indicated by the coefficient α , the size of which can be empirically determined using growth accounting. Variable X_t represents the Harrod neutral technological progress that enhances labor efficiency, while variable A_t represents an exogenous technological shock whose magnitude is either simulated or determined using econometric methods in the case of historical analysis.

The model exhibits its own dynamics of the business cycle, in which the main transmission mechanism of the shock is intertemporal substitution between labor and leisure. In the case of positive exogenous technological progress (variable A), agents supply more labor, leading to higher economic growth, and the economy being in its expansion phase. On the other hand, economic recessions are caused by a lower amount of labor supply due to the mentioned intertemporal optimization. Economic crises are thus caused by voluntary unemployment, where agents prefer leisure (variable L in the utility function) over labor (variable N in the production function).

This type of modeling is significantly more immune to Lucas's critique than Okun's law in equation 10. The parameters σ and ρ have a microeconomic nature, deriving from the psychological characteristics of individual agents and are further endogenizable. The parameter μ is fully endogenized. The parameter α may pose a problem, as well as the parameter δ .

²⁶ King & Rebelo (2000), Ambler & Paquet (1994).

Nonetheless, it represents a significant progress compared to the global macroeconomic parameter c in equation 10.

It is interesting that even modern macroeconomic models cannot completely eliminate the problem of Lucas's critique. In the case of macroeconomics, this is a result of the complexity of the subject being studied. Fully endogenized parameters would lead to models that are too complicated²⁷, and as further chapters will show, it is a complex methodological-philosophical question whether the construction of macroeconomic models that are completely immune to Lucas's critique is even possible.

The RBC model meets Friedman's requirement and is testable based on its predictions. The model provides predictions of the first type, where we test whether we can make better predictions in the Granger sense with knowledge of the model than without it. Typical predictions of this type are impulse response functions, where the model predicts the reaction following an exogenous shock. The model also provides testable predictions of the second type, where it can predict current values of variables based on the optimization performed by individual agents.

Empirical tests of the model have been carried out on a number of its specific predictions. One of the obvious predictions is an increase in voluntary unemployment during economic recessions. The model provides a long list of other predictions based on relationships between variables and their expected correlations. The model also predicts levels of autocorrelation of individual variables and their standard deviations²⁸.

Empirical tests of this model have been conducted extensively. The main assumption about the intertemporal substitution transmission mechanism of leisure time and work was not observed in the data. During economic recessions, there is generally no sharp increase in voluntary unemployment, but rather a high increase in involuntary unemployment, which the model does not incorporate at all. In addition to a long list of other predictions that contradict the data, the model rules out real effects of monetary policy, the significance of which cannot be doubted given the subsequent economic developments.

The methodology behind RBC models, however, allowed for a reduction in the impact of Lucas' critique, and so macroeconomists began to further refine these models instead of rejecting them, especially in the 1990s. Perhaps the most important refinement was the inclusion of nominal and other rigidities, which allowed for the simulation of imperfect market clearing, resulting in the models showing real effects of monetary and fiscal policy. RBC models supplemented with these rigidities began to be called New Keynesian DSGE models, which subsequently became the main building block of the mainstream economic movement called New Keynesian economics. These models gradually became the basis for planning and decision-making by major economic institutions such as central banks or finance ministries²⁹.

7 Two Types of Endogenization

Two types of endogenization can be applied to improve the forecasting ability of the model. The first option is to further endogenize the parameters used. In economic literature, this is referred to as "deep parameters," which are parameters that are no longer endogenized within

²⁷ Some economists believe that unchangeable stable parameters can only be obtained through the complete endogenization to neuropsychological phenomena. For more information, see Dean (2013).

²⁸ The tests are for example described in King & Rebelo (2000).

²⁹ For example Smets & Wouters (2004), Brázdik (2020), Aliyev & Bobková & Štork (2014).

the model (although they have not a unchangeable value similar to the coefficient g in equation 9). If instability of one of the deep parameters reduces the forecasting value of the entire model, it is appropriate to proceed with its further endogenization³⁰.

The second possible endogenization is the inclusion of another area that is not included in the model. This step should be taken if sufficient predictive power cannot be achieved without including additional areas. An example is the financial and mortgage sector, the absence of which in DSGE models led to a methodological impossibility for these models to indicate the start of the economic crisis in 2008 and 2009.

Further endogenization of the model, however, brings a complication in the increase of its complexity. The goal of the model should be its ability to predict proportional to the level of its complexity. Building a model that would accurately reflect reality is like creating a map on a one-to-one scale³¹. Each expansion of the model should therefore be carefully weighed, also because solving DSGE models is a complex mathematical and computational problem. Each subsequent expansion of the model further complicates the calculations.

Criticism of an economic model can be divided into two types. The first type is criticism of insufficient endogenization of a parameter or area. The second type of criticism is a rejection of the conceptual and methodological construction of the model itself. In the case of the first type of criticism, the solution is to expand or modify the model. On the other hand in the case of the second type of criticism, the solution is to reject the model as a whole. While the first type of criticism aims to improve the model, the second type aims to replace it.

In the case of DSGE models, both types of criticism are present in the public sphere. While the first type refers, for example, to missing sectors of the economy or unstable parameters crucial for reliable predictions, the second type focuses on excessive complexity and abstraction, difficult applicability by a wider economic audience and so on³². However, it is a complicated question what concept should replace DSGE models, in order to avoid problems with Lucas' criticism and at the same time achieve similar or better predictive abilities.

8 The Difference Between Behavior and Action

So far, we have been dealing with the construction of models and endogenization of parameters in such a way that the model produces the best results with adequate complexity. In the following three chapters, we will focus on phenomena that methodologically seem to prevent macroeconomic models from making completely reliable predictions.

The difference between physics and macroeconomics lies not only in the instability of parameters but also in the fact that physics does not deal with people, but with inanimate nature. Therefore, the study of physics does not involve free human decision-making. Sokol (2010) distinguishes between “behavior” and “action”³³. While behavior is simply a passive response to the external situation, action is an active activity that takes into account the external situation,

³⁰ When assessing the need for further endogenization, in addition to parameter stability, it is also possible to analyze whether the parameter is policy invariant, meaning that it does not change when policies that the model analyzes change. According to some authors, if this condition is met, it is possible to leave the deep parameter unendogenized despite its unstable nature. However, this view is controversial, see Sergi (2018) for more details.

³¹ J. Robinson.

³² Lawrence & Eichenbaum & Trabandt (2018).

³³ The original text Sokol (2010) uses word “chování” for behavior and “jednání” for action. These terms do not have established linguistic translations. Instead of the word “action”, the term “design” can be also used.

but the action itself is decided based on internal judgment, which is also formed on the basis of moral and ideological positions.

While it can be stated that analyzing planets based on their behavior is a sufficient prerequisite for predicting their movements, reducing human actions to mere behavior is essentially reducing humans to mere unfree machines that mechanically react to external stimuli. It can be said that such an approach denies the basic characteristic of human freedom, which is based on the fact that humans act. This problem can be explained in terms of modeling in the spirit of Friedman's essay on the insignificance of assumptions, as long as we are able to effectively predict based on a model that uses this reduction.

It is difficult to build models based on human action instead of behavior. The assumption that people are behaving is already embedded in the concept of "homo economicus" upon which most modern macroeconomics is built. In the presented DSGE models, all agents behave and therefore only react to incoming shocks based on predefined rules. Even sophisticated institutions such as central banks are only reactive behaving agents in DSGE models, usually based on a one-equation rule in which they minimize the deviation from the inflation target and potential output (the so-called central bank reaction function).

The extent to which reducing actions to behavior is a problem for the effectiveness of predicting in modern macroeconomic models is a complex empirical question. Modern macroeconomics tries to show that essentially all individual actions can be reduced to mere behavior (runs on the bank are fully rational when agents suspect that others will do the same, etc.) and that the role of action is negligible mainly due to the aggregation of individual agents.

The essential point is that the fact of acting humans rules out completely reliable and precise predictions in macroeconomic models. Free individuals can make decisions entirely arbitrarily, while the model will always be built on mere assumptions of rules for human behavior. While the movement of planets can be calculated in advance with knowledge of all input conditions, the final decision of people is until the last moment not determined³⁴.

9 The Butterfly Effect and Input Conditions

Relatively common in terms of prediction is the comparison of macroeconomic and meteorological models. In both cases, these are complex models based on rigorous mathematical foundations, which, however, are not capable of giving accurate predictions in the long term. Despite their incredible complexity and computational demand, meteorological models are not able to provide reliable forecasts for more than two weeks, and for longer predictions, climate models must be used, which are not based on weather forecasting, but on the long-term development of climate. These climate models provide a probabilistic idea of average values, and not specific weather events for a particular day (temperature anomalies, precipitation, etc.).³⁵ Despite high demand, especially from the energy sector, more accurate weather predictions for more than two weeks have not been successful.

Let's now compare macroeconomics and meteorology from a methodological perspective. Both fields of study have a large number of extraneous variables and resemble a falling feather in the analysis of free fall from M. Friedman's example in chapter 4. An important difference between

³⁴ However, even the question of whether the future development of the inanimate universe is predetermined and whether a subject with all input information is able to calculate its subsequent development is a long-standing debate in physics. This is the problem of Laplace's determinism in natural sciences (Kratochvíl (2016)).

³⁵ Krishnamurthy (2019).

the two fields of study is the fact that while the subject of meteorology (weather) only behaves, the subject of macroeconomics (people) acts³⁶. Meteorology thus has the advantage of a system that is closer to the mentioned Laplacian determinism.

A significant common methodological characteristic is the strong influence of initial conditions³⁷. In the case of predicting meteorological models, the main source of imprecision is the lack of precise initial conditions. Even though the European ECWMF model receives 800 million observations daily from land, sea, air, and space³⁸, this state is not sufficient for more precise predictions because even a small inaccuracy in the initial conditions has a significant impact on the subsequent development of the entire system. This fact is described by the butterfly effect, where a small inaccuracy in the initial condition leads, as a consequence of subsequent multiplication, to significant changes in the behavior of the entire system³⁹. According to Lorenz (1993), reliable weather forecasting for more than two weeks is fundamentally excluded due to the multiplicative effect of small differences in initial conditions⁴⁰. The subsequent development of meteorology until 2023 has confirmed this view.

The butterfly effect is also a fundamental complication in macroeconomic modeling. Even in the case of macroeconomics, unlikely events have far-reaching consequences. There are so many possible and unlikely events that it is almost certain that some of them will occur over time. This creates a paradoxical situation where macroeconomic models make predictions that do not account for the occurrence of an unlikely but significant event. However, given the high potential number of these events, there is a high probability that one of them will occur. A typical consequence of this phenomenon is the absence of predictions of macroeconomic changes due to the Covid pandemic in the forecasts from 2019. The macroeconomic models could not predict these changes because it was only a very unlikely initial condition with a significant impact on the system's behavior.

The problem with macroeconomics is also the lack of knowledge about seemingly insignificant initial conditions that can lead to significant changes in the behavior of the entire system through a domino effect. This is a multiplicative effect of seemingly insignificant changes that are closer to the original concept of this phenomenon by Lorenz⁴¹.

The butterfly effect is one of the methodological reasons why meteorological or macroeconomic models can never provide precise long-term predictions. Due to the strength of the butterfly effect, these models will never gather the necessary number of initial conditions⁴² to declare that the system will not develop completely differently due to subsequent strong multiplicative effects of potential deviations.

³⁶ For simplicity, let's exclude the effects of human activities on weather and climate.

³⁷ Lorenz (1993).

³⁸ ECMWF (2023).

³⁹ The attractor presented by Lorenz (1963) simulates the impact of very small deviations in the initial conditions, which, due to subsequent multiplication, significantly change the behavior of the system. More in appendix 3.

⁴⁰ Lorenz (1993).

⁴¹ Lorenz (1963).

⁴² To achieve completely accurate predictions, we would have to be able to obtain input conditions of any precision.

10 The problem of politicization of analyzed issues

Similarly to meteorology, macroeconomics is also often compared to epidemiology. Epidemiological models, much like macroeconomic models, face Lucas's critique (see appendix 1), which complicates their predictions. Furthermore, both fields share a subject of study, which is acting humans. They also share another characteristic, which is the highly politicized nature of their subject of study. The optimal level of measures against the spread of coronavirus or appropriate tax burden, unlike the trajectory of planetary movement or confirmation of the theory of relativity, is a political question.

The cause of this phenomenon is the fact that different measures against the spread of disease or different levels of tax burden do not deal with Pareto-optimal changes, but rather lead to redistribution among people, where every decision generates both winning and losing groups. Questions that do not seek Pareto-optimal solutions are normative in nature, which complicates the way to find general conclusions. Authors often seek conclusions in line with their normative views, and citations are transferred to individual works based on socially prevailing currents of opinion.

The effectiveness of the methodology of natural sciences such as physics is therefore not entirely transferable to areas where the effect of politicization of the studied problem occurs. If the question of the degree of the precession of the planet Neptune's perihelion became an important political issue, the amount of noise around the answers would increase, and the scientific community may never reach a universally accepted result.

One possible solution to this problem is a methodology in which epidemiology or macroeconomics focus exclusively on positive questions - instead of finding appropriate measures only to simulating the impacts of individual scenarios. However, this way of working brings other complications⁴³, where epidemiological or macroeconomic questions would be decided by people who do not have sufficient theoretical knowledge of the subject.

11 Conclusion

This text aims to answer the question of how much the current state of macroeconomic modeling is in line with M. Friedman's essay on the methodology of positive economics from 1966. The second chapter showed that some of the macroeconomic models currently in use do not make forecasts, but instead provide estimates of quantities whose accuracy cannot be verified in the future (such as the mentioned equilibrium exchange rate). These models would undoubtedly fail Friedman's requirement for testing predictions. The second chapter also showed that the predictive relationship itself can be not sufficient to confirm true causality, and complete disregard for assumptions can lead to incorrect conclusions in modern models.

The following chapters illustrated that the complication of macroeconomic models compared to natural sciences is the instability of the parameters used. This issue has complicated forecasting and led to Lucas's critique in the 1980s. Economists developed dynamic stochastic general equilibrium (DSGE) models in response to Lucas's critique and to meet the demand for accurate predictions from the private and public sectors, as well as to establish macroeconomics as a positive science. The effort for reliable prediction is entirely in line with Friedman's ideas,

⁴³ It is useful to add that most of M. Friedman's work also focused on answering normative questions. For example Friedman (1955) or Friedman & Snowden (2002).

but reducing the impact of Lucas's critique led to high mathematical and computational complexity of the models, and the issue of further endogenization is still an open question.

As the work further showed, Friedman's call for reliable predictions faces further methodological complications. Because people do not just react to external conditions, but act as free beings, macroeconomic models that describe only reactive behavior will never be able to provide entirely reliable predictions. The same applies to the strong influence of the butterfly effect, which, when constructing a macroeconomic model, makes it impossible to capture all initial conditions that may play a significant role in subsequent multiplication. The situation is further complicated by the politicization and normative nature of the issues being solved, which complicates the scientific consensus-finding.

Although the effort to construct macroeconomics as a positive science focused on reliable prediction has been widely accepted, this goal has still not been achieved (as illustrated, for example, by the inability of macroeconomic models to predict the financial crisis in 2008). As this paper has shown, for methodological-philosophical reasons, this goal is likely to never be achieved.

Appendix 1

As stated in chapter 10, epidemiological models also incorporate Lucas's critique. For example, using the SEIR model, which is still the most widely used analytical tool for analyzing epidemics, this phenomenon and its impacts can be well illustrated. Consider a population of size N , consisting of susceptible (S), infected (I), and recovered (R) individuals. The numbers of individuals in these groups develop according to the following equations.

$$S_{t+1} = S_t - c\beta S_t I_t \quad (16)$$

$$I_{t+1} = c\beta S_t I_t - \gamma I_t \quad (17)$$

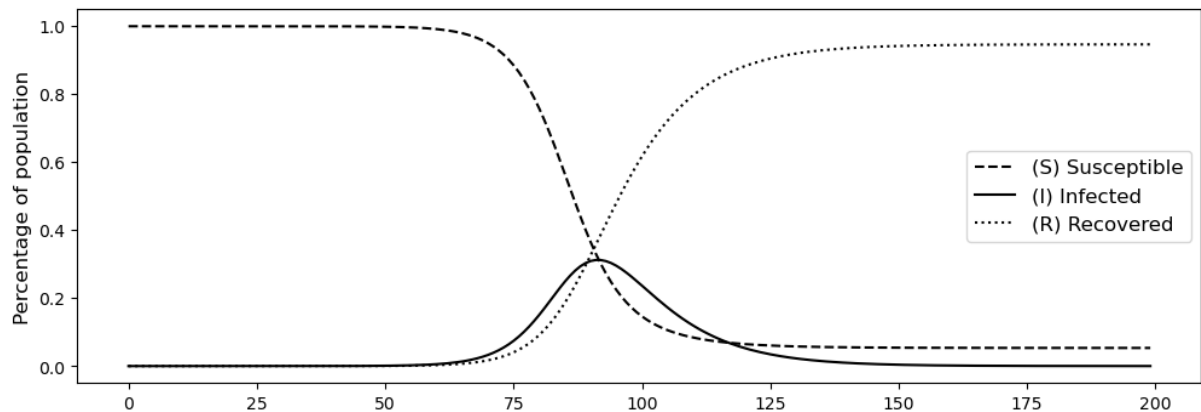
$$R_{t+1} = R_t + \gamma I_t \quad (18)$$

Where the coefficient c represents the number of contacts, the coefficient β denotes the probability of disease transmission when an infected individual comes into contact with a susceptible one, and the coefficient γ indicates the rate at which an infected individual recovers from the disease (and becomes immune to it thereafter)⁴⁴. If the first infected individual appears in the population, the development of the disease in society can be described by figure 1, using the coefficients $c=3$, $\beta=0.1$, and $\gamma=0.1$.⁴⁵

⁴⁴ Calibration of parameters for specific diseases in Taghizadeh & Djafari (2022).

⁴⁵ For simplicity of explanation, let us use a series of simplifications, such as omitting the incubation period or categorizing cases into those who have recovered and those who have passed away.

Figure 1. The basic form of the SEIR model

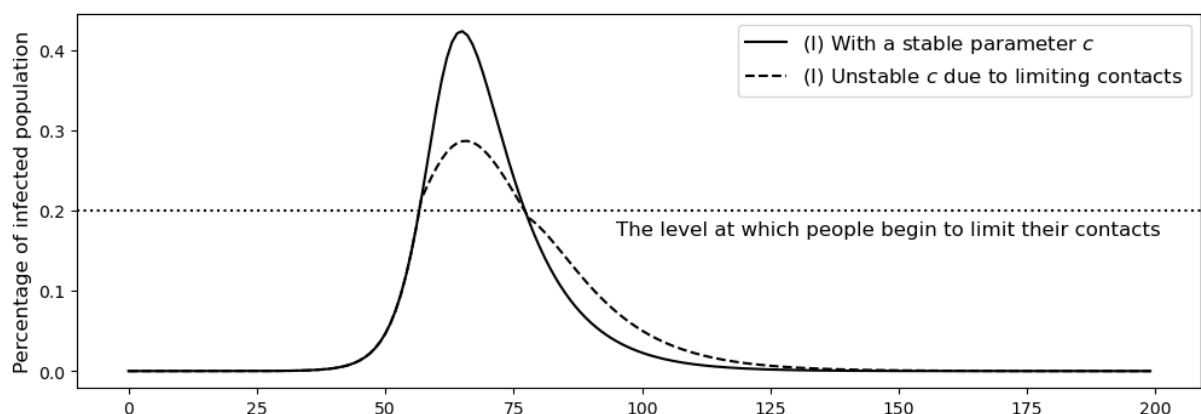


Source: processed by author

The trajectory of the epidemic in the SEIR model shown in figure 1 is entirely dependent on the stability of the coefficients listed above. To illustrate this fact, let us now consider the following situation. If the percentage of the infected population exceeds 20% at a given moment, people will begin to be concerned and will reduce the number of their contacts to 60%. Only after a period of time, when the total number of infected individuals in the population falls below this level, will people return to their original level of contacts.

Such a phenomenon has a fundamental impact on the development of the SEIR model shown in figure 1. The parameter c becomes unstable when the threshold of 20% of the infected population is surpassed, leading to a decrease in the parameter. Let us now compare the development of the number of infected individuals (curve I) and simulate the different developments with stable and unstable parameter c . This simulation is shown in figure 2.

Figure 2. Development of variable I as a function of parameter c



Source: processed by author

On the graph, we can observe the typical impact of Lucas' critique. If the forecast neglects the possible change in the number of contacts, it can yield poor results. Therefore, authors of epidemiological models must work with additional endogenizations and solve problems described in chapter 7, as is the case with macroeconomic models. Modeling in epidemiology either implicitly assumes stable parameters, further endogenizes them or simulates the impact of their changes (for example, simulating the impact of a lockdown, which reduces the number

of contacts c , or a new mutation that increases the infectiousness β), always based on assumptions about subsequent human reactional behavior.

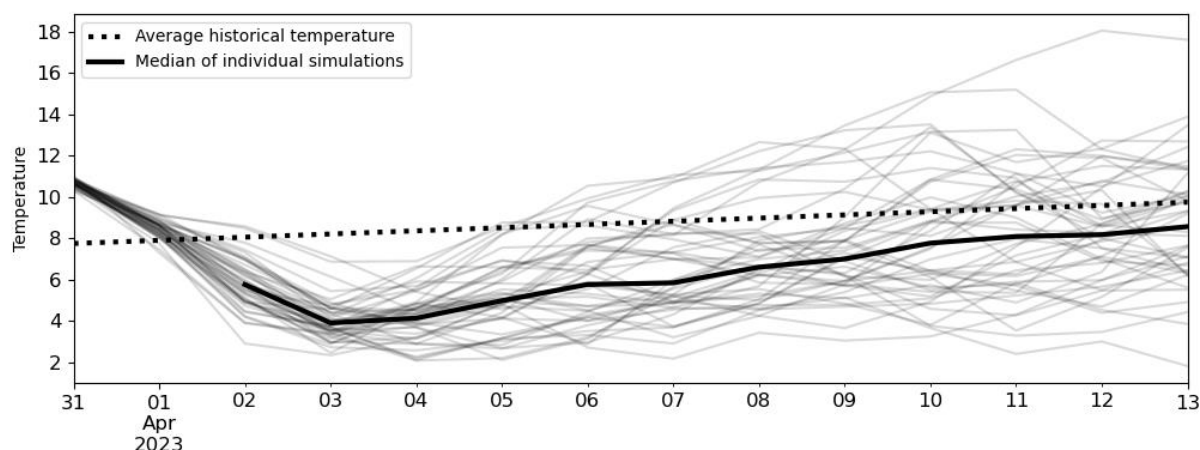
Appendix 2

We can demonstrate the impact of the butterfly effect on real-world data by illustrating how small differences in initial conditions can have significant consequences. For this purpose, let's use weather forecasts used in the energy financial market. Let's take the forecast for Germany from a randomly selected day produced by the ECMWF meteorological model⁴⁶.

First, let's show the temperature forecast. The model begins its calculation by processing the input conditions that determine the current state of the atmosphere. For this purpose, it gathers over 800 million observations from observation stations on land, sea, air and space. Subsequently, based on these input conditions, the supercomputer calculates the forecast using physical rules for weather development.

After the first forecast is computed, the model slightly modifies the input conditions and simulates the weather development again. This step is repeated fifty times, and the output of these simulations with slightly adjusted input conditions is plotted in gray lines in figure 3. The graph shows a similar development for all input conditions in the first two days of the forecast. However, for the following days, the forecasts start to differ significantly.

Figure 3. Two-Week Temperature Forecast

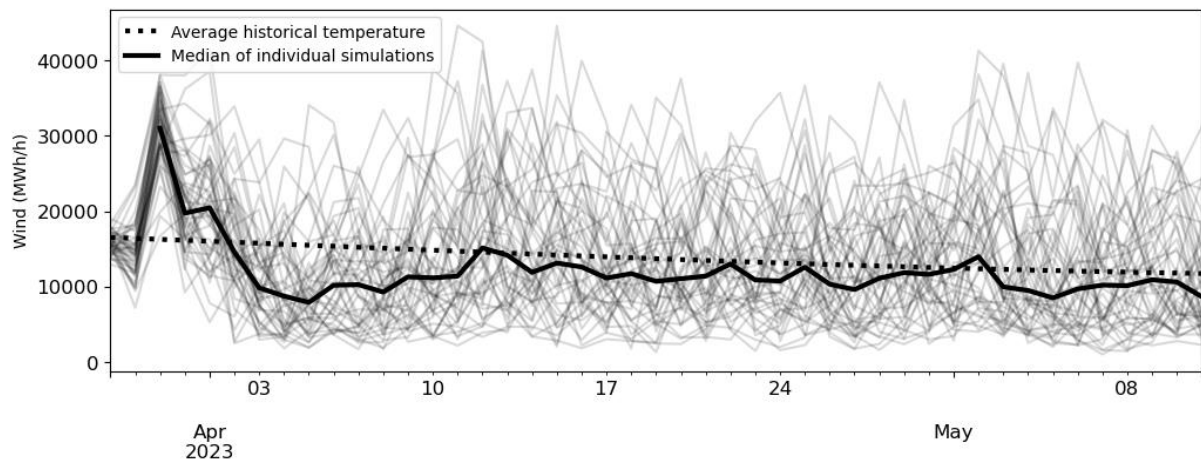


Source: processed by author, data source Refinitiv

After fourteen days, the graph shows that a small difference in the input conditions leads to significant differences in the forecasts (in this case, forecasts differing by more than sixteen degrees). While one of the most extreme forecasts signals highly above-average temperatures, the other signals highly below-average temperatures. As the main forecast, the median of the individual simulations is subsequently used. To capture the degree of uncertainty in the forecast, an analysis of their differences is used, most commonly captured by the percentile range of individual forecasts.

⁴⁶ The financial sector essentially uses only two models, the European model ECMWF and the American model GFS.

Figure 4. Monthly Wind Forecast



Source: processed by author, data source Refinitiv

In the European energy sector, wind strength forecasts are also crucial in addition to temperature. Sensitivity to input deviations is even higher for these forecasts. Let's now use a longer one-month forecast for illustration⁴⁷. The procedure for constructing wind forecasts is the same as in the previous case. Figure 4 shows the development of this system⁴⁸. It is apparent that the input conditions significantly affect the wind forecast, with a range between gale force and calm conditions after just a few days. In the case of temperature, wind, and other predicted variables, the median of simulations always converges to historically average values after one to two weeks. Therefore, for longer-term forecasts, climate models mentioned in chapter 9 are used, which are capable of refining the expected average value.

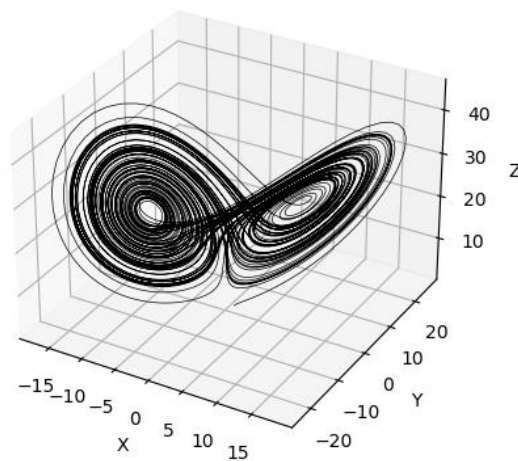
Appendix 3

We illustrated the impact of initial conditions on predicting weather. Let us now demonstrate the concept on which this principle was illustrated by Lorenz (1963) in his original paper, where he describes the concept of deterministic chaos on his attractor.

⁴⁷ The one-month forecast is significantly more demanding computationally and is therefore performed by the ECMWF model only twice a week, while two-week forecasts are computed twice a day.

⁴⁸ The value for wind on the y-axis in the graph is converted to wind power output for the sake of clarity.

Figure 5. Lorenz attractor



Source: processed by author, calculations based on Lorenz (1963)

Deterministic chaos is a system in which the development is pre-defined, and with precise knowledge of the initial conditions, its development can be calculated. However, deterministic chaos is strongly dependent on initial conditions.

Lorenz's attractor is a relatively simple system of three following differential equations

$$\frac{dx}{dt} = \sigma(y - x) \quad (19)$$

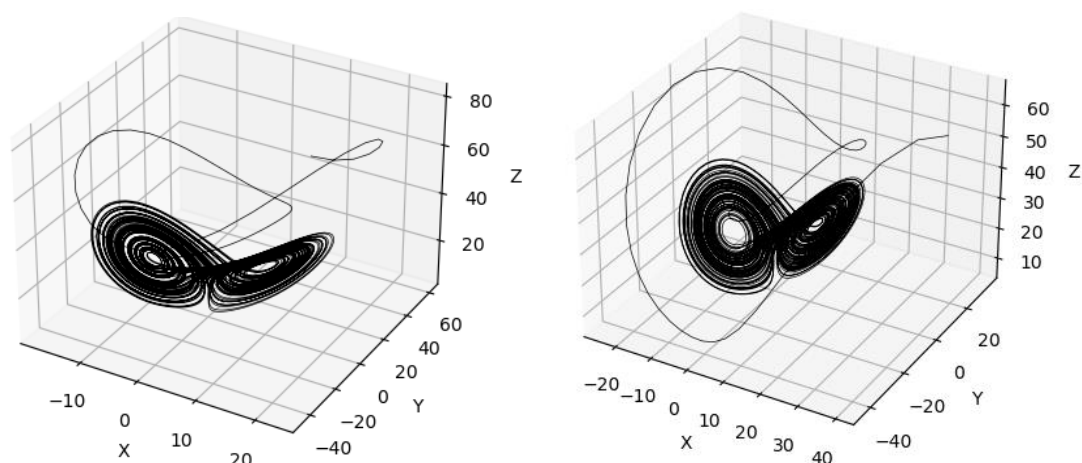
$$\frac{dy}{dt} = x(\rho - z) - y \quad (20)$$

$$\frac{dz}{dt} = xy - \beta z \quad (21)$$

where x , y , and z are the values on axes, and σ , ρ , and β are predefined fixed parameters. The development of such a relationship can be calculated in advance, given knowledge of the input conditions, on which the development heavily depends.

Let us now illustrate this development on graphs. Let's define the fixed parameters $\sigma=8$, $\rho=25$, and $\beta=2.6$. Figure 5 shows the development of the system with input conditions $x_0, y_0, z_0=[1,1,1]$ (the attractor typically exhibits a shape resembling butterfly wings).

Figure 6. Lorenz attractors with various initial conditions



Source: processed by author, calculations based on Lorenz (1963)

Figure 6 shows the development of the system with input conditions $x_0, y_0, z_0 = [5, 60, 56]$ (left side) and $x_0, y_0, z_0 = [40, 35, 60]$ (right side). These graphs show that, in the case of different initial conditions, the curves have completely different values and exhibit a different shape.

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